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On flow and heat transfer in a thin liquid film over an unsteady stretching sheet with variable fluid properties and radiation

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Abstract

In ref. [1] the authors claimed that there are two values of the film thickness. Also, the authors found that the local skin-friction coefficient independent on the viscosity parameter. In the present study it is derived that the film thickness is equal to one. Also, the local skin-friction coefficient is found to depend on the viscosity parameter. It is found that the unsteadiness parameter has one value, which satisfy the constraint condition, for each value of the various parameters when the others are fixed.

Keywords

Liquid Film, Unsteady Stretching Sheet, Thermal Radiation, Variable Fluid Properties

1. Introduction

In recent years the problem of flow and heat transfer within a thin liquid film has received considerable attention by many authors [2- 17] due to its many theoretical and technical applications in the engineering and technology fields. Examples of these applications include wire and fiber coating, reactor fluidization, polymer processing, food stuff processing, transpiration cooling, etc.

The published paper by Liu and Megahed [1] presents the effects of variable viscosity, variable thermal conductivity and thermal radiation on the flow and heat transfer in a thin liquid film over an unsteady stretching sheet. There are many fundamental errors.

2. Mathematical Analysis

Consider the flow in a thin liquid film of uniform thickness $h(t)$ lies on a horizontal elastic sheet, which issues from a slit at the origin of a Cartesian coordinate system as shown in Fig. 1. The $x-$axis is directed along the stretching sheet with the slit at the origin and the $y-$axis normal to the sheet in the outward direction toward the fluid. The fluid motion within the film arises due to the stretching of the elastic sheet.

The velocity and temperature fields in the thin liquid layer are governed by the two-dimensional boundary layer equations for mass, momentum and energy:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 , \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \quad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \quad (3)$$

where u and v are the velocity components in the $x-$ and $y-$directions, respectively. ρ is the fluid density, μ is the fluid viscosity, k is the thermal conductivity, c_p is the specific heat at constant pressure, T is the temperature of the fluid and q_r is the radiation heat flux.

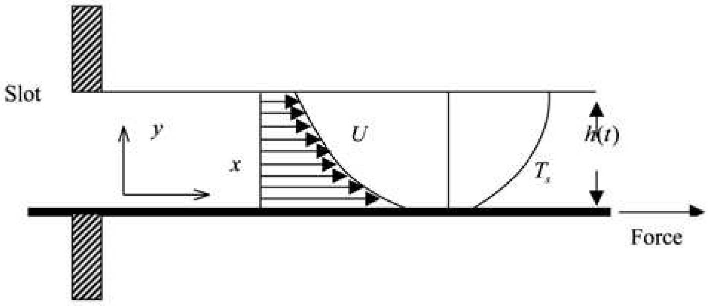

Fig. 1. *Schematic of the physical system.*

The boundary conditions are :

$$u = U(x,t),\ v = 0,\quad T = T_s(x,t)\ \text{ at }\ y = 0\,, \tag{4}$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0\ \text{ at }\ y = h\,, \tag{5}$$

$$v = \frac{dh}{dt}\ \text{ at }\ y = h \tag{6}$$

The sheet moves in its own plane with the velocity [17]:

$$U = \frac{bx}{1-at}, \tag{7}$$

where b and a are positive constants with dimension reciprocal time. The surface temperature of the stretching sheet varies with the distance x and t in the form [7]:

$$T_s = T_0 - T_{ref}\frac{dx^2\rho}{2\mu_\infty}(1-at)^{-\frac{3}{2}}, \tag{8}$$

where T_0 is the temperature at the slit and T_{ref} is a reference temperature. It is noticed that the expressions (7) and (8) are valid only for time $t < \frac{1}{a}$. The radiative heat flux is employed according to Rosseland approximation such that:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{9}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Following Raptis [18], we assume that the temperature difference within the flow are small such that may be expressed as a linear function of the temperature. Expanding in a Taylor series about and neglecting higher-order terms, we have

$$T^4 \cong 4T_0^3 T - 3T_0^4\,. \tag{10}$$

Proceeding with the analysis, we introduce the following transformations [1]:

$$\eta = \left(\frac{b}{\nu}\right)^{\frac{1}{2}}(1-at)^{-\frac{1}{2}}\beta^{-1}y\,, \tag{11}$$

$$u = \frac{bx}{(1-at)}f'(\eta), \tag{12}$$

$$v = -\left(\frac{\mu_\infty b}{\rho}\right)\left(\frac{1}{1-at}\right)\beta f(\eta), \tag{13}$$

$$T = T_0 - T_{ref}\frac{dx^2\rho}{2\mu_\infty}(1-at)^{-\frac{3}{2}}\theta(\eta), \tag{14}$$

where f is the dimensionless stream function, θ is the dimensionless temperature of the fluid . β is yet an unknown constant denoting the dimensionless film thickness which can be defined as [1]:

$$\beta = \left(\frac{b\rho}{\mu_\infty}\right)^{\frac{1}{2}}(1-at)^{-1/2}h(t)\,. \tag{15}$$

The temperature dependant viscosity and thermal conductivity are given as [1]:

$$\mu = \mu_o e^{-\alpha\theta}, \tag{16}$$

$$\kappa = \kappa_o(1+\varepsilon\theta), \tag{17}$$

where α is the viscosity parameter and ε is the thermal conductivity parameter, μ_o and κ_o are the ambient viscosity and thermal conductivity, respectively. Using the transformations (11)-(14) with Eqs.(9),(10),(16)and(17), the governing equations (1)-(3) and the boundary conditions(4)-(6) becom:

$$e^{\alpha\theta}(f''' + \alpha\theta' f'') + \gamma[f f'' - f'^2 - S(\frac{\eta}{2}f'' + f')] = 0\,, \tag{18}$$

$$(1+R+\varepsilon\theta)\theta'' + \varepsilon\theta'^2 + P_r\gamma[(f\theta' - 2f'\theta) - S(\frac{\eta}{2}\theta' + \frac{3}{2}\theta)] = 0, \tag{19}$$

$$f = 0\,,\ f' = 1\,,\ \theta = 1 \qquad at\ \eta = 0\,, \tag{20}$$

$$f'' = 0\,, \theta' = 0 \qquad at\ \eta = 1\,, \tag{21}$$

$$f = \frac{s}{2} \qquad at\ \eta = 1\,, \tag{22}$$

where a prime denotes differentiation with respect to η, $s = \dfrac{a}{b}$ is the unsteadiness parameter, $P_r = \dfrac{\mu_o c_p}{\kappa_o}$ is the Prandtl number, $\gamma = \beta^2$ and $R = \dfrac{16\sigma^* T_o^3}{3k\kappa^*}$ is the radiation parameter.

3. Comments on the Published Paper " Journal of Mechanics / Volume 28 / Issue 02 / June 2012, pp 291 297" by IC.Liu and A. M. Megahed

In this comment we should use the same symbols and equations defined in ref. [1].

1. In ref. [1], the dimensionless film thickness has two values: (i) equal to β (below Eq.(14)) and (ii) equal to $\gamma = \beta^2$ also (below Eq.(22)). This is not correct because the film thickness is unique.

2. The authors defined in ref. [1] the dimensionless film thickness β as:

$$\beta = (\frac{\rho\, b}{\mu_0})^{1/2}(1-at)^{-1/2} h(t).$$

This definition obtained as the value of $\eta = (\frac{\rho\, b}{\mu_0})^{1/2}(1-at)^{-1/2} y$ at $y = h$. While the similarity variable η in ref. [1] is given as:

$$\eta = (\frac{\rho\, b}{\mu_0})^{1/2}(1-at)^{-1/2}\ \beta^{-1} y\,,$$

which leads to that the film thickness equal to 1 as following:

Using Eq. (15) in Eq. (11), then $\eta = \frac{y}{h}$, where $h(t)$ is the thickness of the liquid film. At $y = h$, the dimensionless thickness film equal to 1. At this point β does not represent the film thickness.

Using Eqs. (9)-(17), Eqs. (1)-(3) with the boundary conditions (4)-(6) become :

$$e^{\alpha\theta}(f''' + \alpha\theta' f'') + \frac{\rho\, b h^2}{\mu_0(1-at)}(ff'' - f'^2 - S(f' + \frac{\eta}{2} f'')) = 0,$$

$$\frac{1}{Pr}((1 + R + \varepsilon\theta)\theta'' + \varepsilon\theta'^2) + \frac{\rho\, b h^2}{\mu_0(1-at)}(f\theta' - 2f'\theta - S(\frac{3}{2}\theta + \frac{\eta}{2}\theta')) = 0,$$

$$f = 0, \quad f' = 1, \quad \theta = 1 \quad at \quad \eta = 0,$$

$$f'' = 0, \quad \theta' = 0, \quad f = \frac{S}{2} \ \text{at}\ \eta = 1.$$

The similarity solution exists only when $h = (\frac{b}{\mu_0/\rho})^{1/2}(1-at)^{-1/2}$, then, the above transformed equations become :

$$e^{\alpha\theta}(f''' + \alpha\theta' f'') + (ff'' - f'^2 - S(f' + \frac{\eta}{2} f'')) = 0,$$

$$\frac{1}{Pr}((1 + R + \varepsilon\theta)\theta'' + \varepsilon\theta'^2) + (f\theta' - 2f'\theta - S(\frac{3}{2}\theta + \frac{\eta}{2}\theta')) = 0,$$

$$f = 0, \quad f' = 1, \quad \theta = 1 \quad at \quad \eta = 0,$$

$$f'' = 0, \quad \theta' = 0, \quad f = \frac{S}{2} \ \text{at}\ \eta = 1.$$

There is one fixed value of the unsteadiness parameter S which satisfy the constraint equation $f = S/2 \ \ at \ \ \eta = 1$ as shown in Table 1.

Then, the transformed equations (18) and (19) in ref.[1] are wrong . Consequently the results obtained by Liu and Megahed [1] are wrong. This is common error exists in many published papers [19-22].

3. In ref. [1] the wall shear stress τ_w and the surface heat transfer q_w defined by Eq. (24) are wrong, (consequently Eq.(25) and (26) are wrong). The corrected forms of τ_w and q_w are :

$$\tau_w = -(\mu \frac{\partial u}{\partial y})_{y=0} = -\mu_0 e^{\alpha\theta(0)}(\frac{\partial u}{\partial y})_{y=0},$$

$$q_w(x,t) = -(\kappa \frac{\partial T}{\partial y})_{y=0} = -\kappa_o(1 + \varepsilon\theta(0) + R)(\frac{\partial T}{\partial y})_{y=0}.$$

then the local skin-friction coefficient Cf_x (not Cf) in Eq.(25) (see ref. [1]) is wrong and the correct form of Cf_x is :

$$Cf_x = \frac{2\tau_w}{\rho U^2} = -2\, Re_x^{1/2}\, e^{\alpha\theta(0)} f''(0).$$

This error exists in many published papers [23-27].

From the above definition it noticed that the local skin-friction coefficient Cf_x depends on the viscosity parameter α in explicit form and on the thermal conductivity parameter ε, thermal radiation parameter R and the Prandtl number Pr in implicit form .

The results obtained for the local skin-friction coefficient in terms of $e^{\alpha\theta(0)} f''(0)$, the local Nusselt number and the free surface temperature is given in Table 1. The effect of α on the local skin-friction (in terms of $-e^{-\alpha\theta(0)} f''(0)$ not $-f''(0)$) has an opposite effect than that obtained in ref. [1].

4. If β is the film thickness (as in ref. [1]), it is noticed that the following contradiction in Tables 2-6 (in ref. [1]):

The free surface temperature $\theta(1)$ obtained at $y = h(\eta = 1)$. This is wrong because the free surface temperature is the temperature at the value of the film thickness, i.e. $\theta(\beta)$.

Table 1. *The values of the local skin-friction coefficient, the local Nusselt number and the free surface temperature for various value of parameters.*

α	ε	R	Pr	**S**	$-e^{\alpha\theta(0)}f''(0)$	$-\theta'(0)$	$\theta(1)$
0	0.1	1	1	1.27689	1.26831	1.07263	0.54568
0.1				1.30287	1.31582	1.08687	0.53945
0.2				1.32843	1.36458	1.10070	0.53343
0.4				1.37823	1.46143	1.12709	0.52197
0.1	0	1	1	1.30270	1.31593	1.12847	0.52904
	0.1			1.30287	1.31632	1.08687	0.53945
	0.2			1.30303	1.31660	1.04858	0.54957
	0.6			1.30358	1.31758	0.92124	0.58717
0.1	0.1	0	1	1.30014	1.31157	1.67570	0.34655
		1		1.30287	1.31632	1.08687	0.53945
		2		1.30424	1.31871	0.81079	0.64473
		3		1.30508	1.32017	0.64795	0.71084
0.1	0.1	1	0.7	1.30413	1.31852	0.83142	0.63571
			1	1.30287	1.31632	1.08687	0.53945
			2	1.29995	1.31124	1.73205	0.33798
			3	1.29809	1.30798	2.20818	0.22939

4. Conclusion

In the present work, the effects of variable fluid properties and thermal radiation on the flow and heat transfer within a thin liquid film have been studied. The governing fundamental equations are transformed to a system of nonlinear ordinary differential equations which was solved numerically. From the numerical results, we observed that, the thin film thickness equal to 1 and independent on α, ε, R and Pr. Also, it was found that the unsteadiness parameter S has a fixed value for any value of parameter when the others are fixed. In addition, the local skin-friction coefficient increases with increasing the viscosity parameter and the thermal conductivity parameter. However, it noticed that the thermal radiation parameter has the effect of decreasing the local Nusselt number and enhancing the local skin – friction coefficient. Finally, the local Nusselt number increases with increasing the viscosity parameter while it decreases with increasing the thermal conductivity parameter.

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